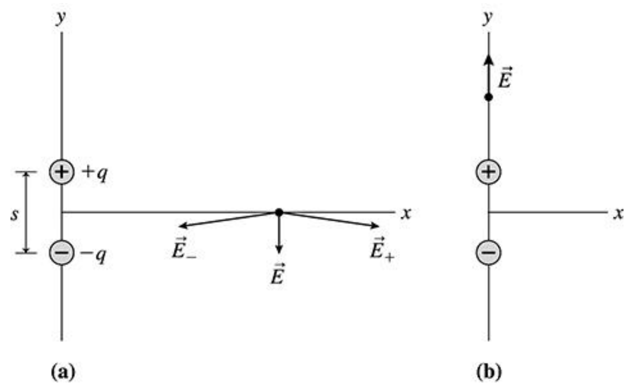
27.12. $E_1 = E_2 = E_3 = E_4 = E_5$. The electric field is constant everywhere between the plates. This is indicated by the electric field vectors, which are all the same length and in the same direction.

27.5. Model: The distances to the observation points are large compared to the size of the dipole, so model the field as that of a dipole moment.

Visualize:



The dipole consists of charges $\pm q$ along the y-axis. The electric field in (a) points down. The field in (b) points up.

Solve: (a) The dipole moment is

$$\vec{p} = (qs,\ \text{from} - \text{to} +) = (1.0\times10^{-9}\ \text{C})(0.0020\ \text{m})\,\hat{j} = 2.0\times10^{-12}\,\hat{j}\ \text{C m}$$

The electric field at (10 cm, 0 cm), which is at distance $r = 0.10$ m in the plane perpendicular to the electric dipole, is

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3} = -(9.0\times10^9\ \text{N m}^2/\text{C}^2)\frac{2.0\times10^{-12}\,\hat{j}\ \text{C m}}{(0.10\ \text{m})^3} = -18.0\,\hat{j}\ \text{N/C}$$

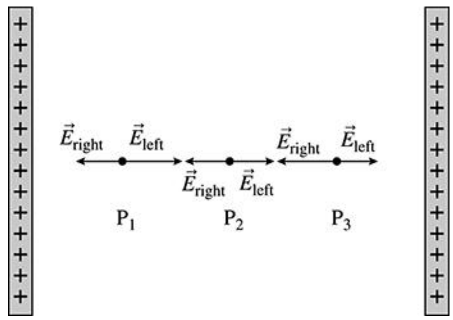
The field strength, which is all we're asked for, is 18.0 N/C.

(b) The electric field at (0 cm, 10 cm), which is at $r = 0.10$ m along the axis of the dipole, is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3} = (9.0\times10^9\ \text{N m}^2/\text{C}^2)\frac{2(2.0\times10^{-12}\,\hat{j}\ \text{C m})}{(0.10\ \text{m})^3} = 36\,\hat{j}\ \text{N/C}$$

The field strength at this point is 36 N/C.

27.9. Model: The rods are thin. Assume that the charge lies along a *line*.
Visualize:



Because both the rods are positively charged, the electric field from each rod points away from the rod. Because the electric fields from the two rods are in opposite directions at P_1, P_2, and P_3, the net field strength at each point is the difference of the field strengths from the two rods.

Solve: Example 27.3 gives the electric field strength in the plane that bisects a charged rod:

$$E_{\text{rod}} = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r\sqrt{r^2+(L/2)^2}}$$

The electric field from the rod on the right at a distance of 1 cm from the rod is

$$E_{\text{right}} = (9.0\times10^9 \text{ N m}^2/\text{C}^2)\frac{10\times10^{-9}\text{ C}}{(0.01\text{ m})\sqrt{(0.01\text{ m})^2+(0.05\text{ m})^2}} = 1.765\times10^5\text{ N/C}$$

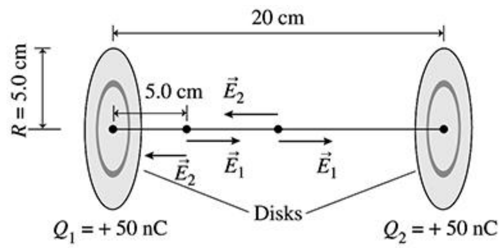
The electric field from the rod on the right at distances 2 cm and 3 cm from the rod are 0.835×10^5 N/C and 0.514×10^5 N/C. The electric fields produced by the rod on the left at the same distances are the same. Point P_1 is 1.0 cm from the rod on the left and is 3.0 cm from the rod on the right. Because the electric fields at P_1 have opposite directions, the net electric field strengths are

At 1.0 cm $\quad E = 1.765\times10^5\text{ N/C} - 0.514\times10^5\text{ N/C} = 1.25\times10^5\text{ N/C}$

At 2.0 cm $\quad E = 0.835\times10^5\text{ N/C} - 0.835\times10^5\text{ N/C} = 0\text{ N/C}$

At 3.0 cm $\quad E = 1.765\times10^5\text{ N/C} - 0.514\times10^5\text{ N/C} = 1.25\times10^5\text{ N/C}$

27.14. Model: Model each disk as a uniformly charged disk. When the disk is positively charged, the on-axis electric field of the disk points away from the disk.
Visualize:



Solve: **(a)** The surface charge density on the disk is

$$\eta = \frac{Q}{A} = \frac{Q}{\pi R^2} = \frac{50\times10^{-9}\ \text{C}}{\pi(0.050\ \text{m})^2} = 6.366\times10^{-6}\ \text{C/m}^2$$

From Equation 27.23, the electric field of the left disk at $z = 0.10$ m is

$$(E_1)_z = \frac{\eta}{2\varepsilon_0}\left[1-\frac{1}{\sqrt{1+R^2/z^2}}\right] = \frac{6.366\times10^{-6}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N m}^2)}\left[1-\frac{1}{\sqrt{1+(0.050\ \text{m}/0.10\ \text{m})^2}}\right] = 38{,}000\ \text{N/C}$$

Hence, $\vec{E}_1 = (38{,}000\ \text{N/C, right})$. Similarly, the electric field of the right disk at $z = 0.10\,\text{m}$ (to its left) is $\vec{E}_2 = (38{,}000\ \text{N/C, left})$. The net field at the midpoint between the two disks is $\vec{E} = \vec{E}_1 + \vec{E}_2 = 0\ \text{N/C}$.
(b) The electric field of the left disk at $z = 0.050$ m is

$$(E_1)_z = \frac{6.366\times10^{-6}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N m}^2)}\left[1-\frac{1}{\sqrt{1+(0.050\ \text{m}/0.10\ \text{m})^2}}\right] = 1.05\times10^5\ \text{N/C} \Rightarrow \vec{E}_1 = (1.05\times10^5\ \text{N/C, right})$$

Similarly, the electric field of the right disk at $z = 0.15$ m (to its left) is $\vec{E}_2 = (1.85\times10^4\ \text{N/C, left})$. The net field is thus

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = (8.7\times10^4\ \text{N/C, right})$$

The field strength is 8.7×10^4 N/C.

27.19. Model: The electric field in a region of space between the plates of a parallel-plate capacitor is uniform.
Solve: The electric field inside a capacitor is $E = Q/\varepsilon_0 A$. Thus, the charge needed to produce a field of strength E is

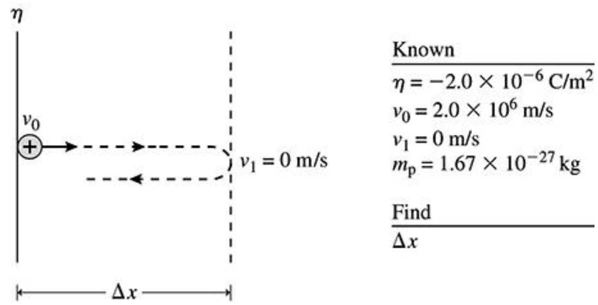
$$Q = \varepsilon_0 AE = (8.85\times10^{-12}\ \text{C}^2/\text{N m}^2)\left[\pi(0.020\ \text{m})^2\right](3.0\times10^6\ \text{N/C}) = 33.4\ \text{nC}$$

The number of electrons transferred from one plate to the other is

$$\frac{33.4\times10^{-9}\ \text{C}}{1.60\times10^{-19}\ \text{C}} = 2.1\times10^{11}$$

27.24. Model: The infinite negatively charged plane produces a uniform electric field that is directed toward the plane.

Visualize:



Solve: From the kinematic equation of motion $v_1^2 = 0 = v_0^2 + 2a\Delta x$ and $F = qE = ma$,

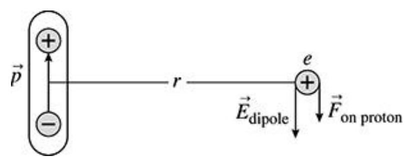
$$a = \frac{qE}{m} = \frac{-v_0^2}{2\Delta x} \Rightarrow \Delta x = \frac{-mv_0^2}{2qE}$$

Furthermore, the electric field of a plane of charge with surface charge density η is $E = \eta/2\varepsilon_0$. Thus,

$$\Delta x = \frac{-mv_0^2\varepsilon_0}{q\eta} = \frac{-\left(1.67\times10^{-27}\text{ kg}\right)\left(2.0\times10^{6}\text{ m/s}\right)^2\left(8.85\times10^{-12}\text{ C}^2/\text{N m}^2\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(-2.0\times10^{-6}\text{ C/m}^2\right)} = 0.185\text{ m}$$

27.27. Model: The size of a molecule is ≈ 0.1 nm. The proton is 2.0 nm away, so $r >> s$ and we can use Equation 27.12 for the electric field in the plane that bisects the dipole.

Visualize:



Solve: You can see from the diagram that $\vec{F}_{\text{dipole on proton}}$ is opposite to the direction of $\vec{p}$. The magnitude of the dipole field at the position of the proton is

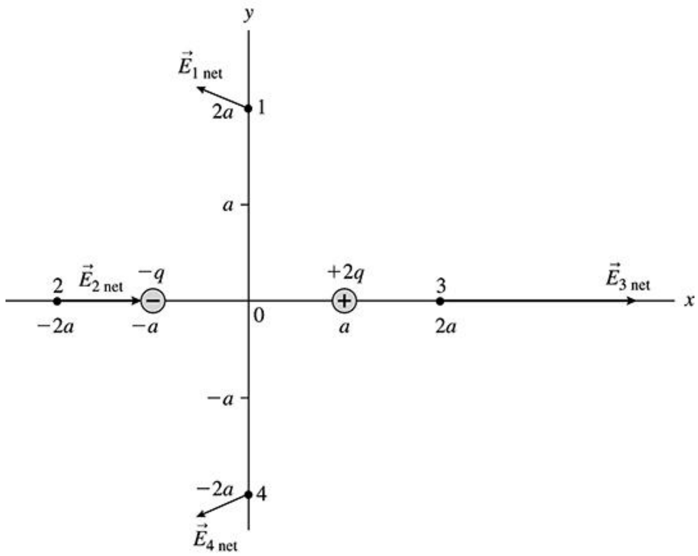
$$E_{\text{dipole}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} = (9.0\times10^{9}\text{ N m}^2/\text{C}^2)\frac{5.0\times10^{-30}\text{ C m}}{(2.0\times10^{-9}\text{ m})^3} = 5.624\times10^{5}\text{ N/C}$$

The magnitude of $\vec{F}_{\text{dipole on proton}}$ is

$$F_{\text{dipole on proton}} = eE_{\text{dipole}} = (1.60\times10^{-19}\text{ C})(5.624\times10^{5}\text{ N/C}) = 9.0\times10^{-13}\text{ N}$$

Including the direction, the force is $\vec{F}_{\text{dipole on proton}} = (9.0 \times 10^{-13}$ N, direction opposite $\vec{p}$).

27.32. Model: The electric field is that of two charges $-q$ and $+2q$ located at $x = \pm a$.
Visualize:



Solve: **(a)** At point 1, the electric field from $-q$ is

$$E_{-q} = \frac{1}{4\pi\varepsilon_0}\frac{|-q|}{(a)^2 + (2a)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{5a^2}$$

$\vec{E}_{-q}$ points toward $-q$ and makes an angle $\phi_1 = \tan^{-1}(2a/a) = 63.43°$ below the $-x$-axis, hence

$$\vec{E}_{-q} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{5a^2}\right)\left(-\cos\phi_1\hat{i} - \sin\phi_1\hat{j}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{5a^2}\right)\left(-\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{5\sqrt{5}a^2}\right)\left(-\hat{i} - 2\hat{j}\right)$$

The electric field from the $+2q$ is

$$E_{+2q} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{a^2 + (2a)^2} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{5a^2}$$

$\vec{E}_{+2q}$ points away from $+2q$ and makes an angle $\phi_1 = \tan^{-1}(2a/a) = 63.43°$ above the $-x$-axis. So,

$$\vec{E}_{+2q} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{5a^2}\right)\left(-\cos\phi_2\hat{i} + \sin\phi_2\hat{j}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{5\sqrt{5}a^2}\right)\left(-2\hat{i} + 4\hat{j}\right)$$

Adding these two vectors,

$$\vec{E}_{1\text{ net}} = \vec{E}_{-q} + \vec{E}_{+2q} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{5\sqrt{5}a^2}\right)\left(-3\hat{i} + 2\hat{j}\right)$$

At point 2, the electric field from $-q$ points toward $-q$, so

$$\vec{E}_{-q} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{a^2}\right)\hat{i}$$

The electric field from $+2q$ points away from $+2q$, so

$$\vec{E}_{+2q} = -\frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{9a^2}\right)\hat{i}$$

Adding these two vectors,

27.43. Model: The electric field is that of a line charge of length L.

Visualize: Please refer to Figure P27.43. Let the bottom end of the rod be the origin of the coordinate system. Divide the rod into many small segments of charge Δq and length $\Delta y'$. Segment i creates a small electric field at the point P that makes an angle θ with the horizontal. The field has both x and y components, but $E_z = 0$ N/C. The distance to segment i from point P is $\left(x^2 + y'^2\right)^{1/2}$.

Solve: The electric field created by segment i at point P is

$$\vec{E}_i = \frac{\Delta q}{4\pi\varepsilon_0\left(x^2 + y'^2\right)}\left(\cos\theta\hat{i} - \sin\theta\,\hat{j}\right) = \frac{\Delta q}{4\pi\varepsilon_0\left(x^2 + y'^2\right)}\left(\frac{x}{\sqrt{x^2 + y'^2}}\hat{i} - \frac{y'}{\sqrt{x^2 + y'^2}}\,\hat{j}\right)$$

The net field is the sum of all the $\vec{E}_i$, which gives $\vec{E} = \sum_i \vec{E}_i$. Δq is not a coordinate, so before converting the sum to an integral we must relate charge Δq to length $\Delta y'$. This is done through the linear charge density $\lambda = Q/L$, from which we have the relationship

$$\Delta q = \lambda \Delta y' = \frac{Q}{L}\Delta y'$$

With this charge, the sum becomes

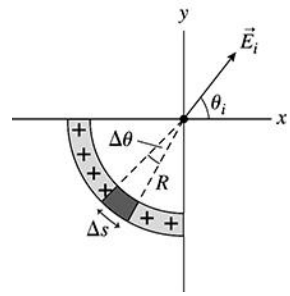
$$\vec{E} = \frac{Q/L}{4\pi\varepsilon_0}\sum_i\left[\frac{x\Delta y'}{\left(x^2 + y'^2\right)^{3/2}}\hat{i} - \frac{y'\Delta y'}{\left(x^2 + y'^2\right)^{3/2}}\,\hat{j}\right]$$

Now we let $\Delta y' \to dy'$ and replace the sum by an integral from $y' = 0$ m to $y' = L$. Thus,

$$\vec{E} = \frac{(Q/L)}{4\pi\varepsilon_0}\left(\int_0^L \frac{xdy'}{\left(x^2 + y'^2\right)^{3/2}}\hat{i} - \int_0^L \frac{y'dy'}{\left(x^2 + y'^2\right)^{3/2}}\,\hat{j}\right) = \frac{(Q/L)}{4\pi\varepsilon_0}\left(x\left[\frac{y'}{x^2\sqrt{x^2 + y'^2}}\right]_0^L \hat{i} - \left[\frac{-1}{\sqrt{x^2 + y'^2}}\right]_0^L \hat{j}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0}\frac{1}{x\sqrt{x^2 + L^2}}\hat{i} \; - \; \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{Lx}\right)\left(1 - \frac{x}{\sqrt{x^2 + L^2}}\right)\hat{j}$$

27.47. Model: Assume that the quarter-circle plastic rod is thin and that the charge lies along the quarter-circle of radius R.
Visualize:



The origin of the coordinate system is at the center of the circle. Divide the rod into many small segments of charge Δq and arc length Δs.

Solve: **(a)** Segment i creates a small electric field $\vec{E}_i$ at the origin with two components:

$$(E_i)_x = E_i \cos\theta_i \qquad\qquad (E_i)_y = E_i \sin\theta_i$$

Note that the angle θ_i depends on the location of the segment i. Now all segments are at distance $r_i = R$ from the origin, so

$$E_i = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{r_i^2} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{R^2}$$

The linear charge density of the rod is $\lambda = Q/L$, where L is the rod's length (L = quarter-circumference = $\pi R/2$). This allows us to relate charge Δq to the arc length Δs through

$$\Delta q = \lambda\Delta s = \left(\frac{Q}{L}\right)\Delta s = \left(\frac{2Q}{\pi R}\right)\Delta s$$

Using $\Delta s = R\Delta\theta$, the components of the electric field at the origin are

$$(E_i)_x = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{R^2}\cos\theta_i = \frac{1}{4\pi_0}\frac{1}{R^2}\left(\frac{2Q}{\pi R}\right)R\Delta\theta\cos\theta_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{\pi R^2}\right)\Delta\theta\cos\theta_i$$

$$(E_i)_y = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{R^2}\sin\theta_i = \frac{1}{4\pi\varepsilon_0}\frac{1}{R^2}\left(\frac{2Q}{\pi R}\right)R\Delta\theta\sin\theta_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{\pi R^2}\right)\Delta\theta\sin\theta_i$$

(b) The x- and y-components of the electric field for the entire rod are the integrals of the expressions in part (a) from
$\theta = 0$ rad to $\theta = \pi/2$. We have

$$E_x = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{\pi R^2}\right)\int_0^{\pi/2}\cos\theta\, d\theta \qquad\qquad E_y = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{\pi R^2}\right)\int_0^{\pi/2}\sin\theta\, d\theta$$

(c) The integrals are

$$\int_0^{\pi/2}\sin\theta d\theta = \left[-\cos\theta\right]_0^{\pi/2} = -\left(\cos\frac{\pi}{2} - \cos 0\right) = +1 \qquad\qquad \int_0^{\pi/2}\cos\theta d\theta = \left[\sin\theta\right]_0^{\pi/2} = \sin\frac{\pi}{2} - \sin 0 = +1$$

The electric field is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{\pi R^2}\left(\hat{i} + \hat{j}\right)$$

27.53. Model: Assume that the electric field inside the capacitor is constant, so constant-acceleration kinematic equations apply.

Visualize: Please refer to Figure P27.53.

Solve: **(a)** The force on the electron inside the capacitor is

$$\vec{F} = m\vec{a} = q\vec{E} \Rightarrow \vec{a} = \frac{q\vec{E}}{m}$$

Because $\vec{E}$ is directed upward (from the positive plate to the negative plate) and $q = -1.60\times10^{-19}$ C , the acceleration of the electron is downward. We can therefore write the above equation as simply $a_y = qE/m$. To determine E, we must first find a_y. From kinematics,

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2 \Rightarrow 0.040 \text{ m} = 0 \text{ m} + v_0 \cos 45°(t_1 - t_0) + 0 \text{ m}$$

$$\Rightarrow (t_1 - t_0) = \frac{(0.040 \text{ m})}{(5.0\times10^6 \text{ m/s})\cos 45°} = 1.1314\times10^{-8} \text{ s}$$

Using the kinematic equations for the motion in the y-direction,

$$v_{1y} = v_{0y} + a_y\left(\frac{t_1 - t_0}{2}\right) \Rightarrow 0 \text{ m/s} = v_0 \sin 45° + \left(\frac{qE}{m}\right)\left(\frac{t_1 - t_0}{2}\right)$$

$$\Rightarrow E = -\frac{2 \text{ m } v_0 \sin 45°}{q(t_1 - t_0)} = -\frac{2(9.1\times10^{-31} \text{ kg})(5.0\times10^6 \text{ m/s})\sin 45°}{(-1.60\times10^{-19} \text{ C})(1.1314\times10^{-8} \text{ s})} = 3550 \text{ N/C} = 3.6\times10^3 \text{ N/C}$$

(b) To determine the separation between the two plates, we note that $y_0 = 0$ m and $v_{0y} = (5.0\times10^6 \text{ m/s})\sin 45°$, but at $y = y_1$, the electron's highest point, $v_{1y} = 0$ m/s. From kinematics,

$$v_{1y}^2 = v_{0y}^2 + 2a_y(y_1 - y_0) \Rightarrow 0 \text{ m}^2/\text{s}^2 = v_0^2 \sin^2 45° + 2a_y(y_1 - y_0)$$

$$\Rightarrow (y_1 - y_0) = -\frac{v_0^2 \sin^2 45°}{2a_y} = -\frac{v_0^2}{4a_y}$$

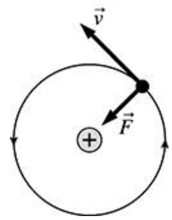
From part (a),

$$a_y = \frac{qE}{m} = \frac{(-1.60\times10^{-19} \text{ C})(3550 \text{ N/C})}{9.1\times10^{-31} \text{ kg}} = -6.242\times10^{14} \text{ m/s}^2$$

$$\Rightarrow y_1 - y_0 = -\frac{(5.0\times10^6 \text{ m/s})^2}{4(-6.242\times10^{14} \text{ m/s}^2)} = 0.010 \text{ m} = 1.0 \text{ cm}$$

This is the height of the electron's trajectory, so the minimum spacing is 1.0 cm.

27.58. Model: The electron orbiting the proton experiences a force given by Coulomb's law.
Visualize:



Solve: The force that causes the circular motion is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_p|q_e|}{r^2} = \frac{m_e v^2}{r} = \frac{m_e\left(4\pi^2 r^2 f^2\right)}{r}$$

where we used $v = 2\pi r/T = 2\pi r f$. The frequency is

$$f = \sqrt{\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_p|q_e|}{4\pi^2 m_e r^3}} = \sqrt{\frac{\left(9.0\times10^9\ \text{N m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{4\pi^2\left(9.11\times10^{-31}\ \text{kg}\right)\left(5.3\times10^{-11}\ \text{m}\right)^3}} = 6.56\times10^{15}\ \text{Hz}$$

27.3. $E_3 = E_4 > E_2 > E_1$. The electric field strength is larger in the region where the field lines are closer together (E_3 and E_4) and smaller where the field lines are farther apart.

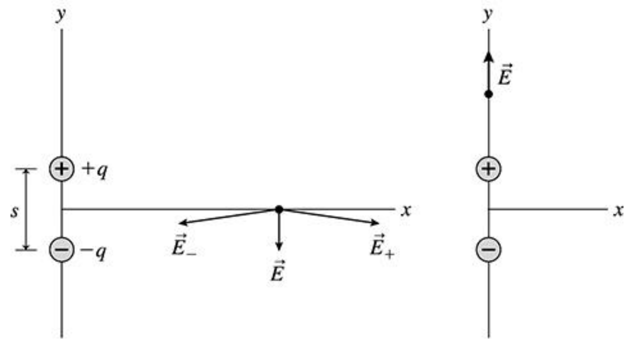
27.8. **(a)** $A_f = \dfrac{A_i}{3.163^2}$, so $\dfrac{N_f}{N_i} = \dfrac{\left(Q/A_f\right)}{\left(Q/A_i\right)} = \dfrac{A_i}{A_f} = 3.163^2 = 10$

(b) $F_f = eE_f = e\dfrac{N_f}{2E_0}$ and $F_i = eE_i = e\dfrac{N_i}{2E_0}$, so $\dfrac{F_f}{F_i} = \dfrac{N_f}{N_i} = 10.$

27.12. $E_1 = E_2 = E_3 = E_4 = E_5$. The electric field is constant everywhere between the plates. This is indicated by the electric field vectors, which are all the same length and in the same direction.

27.5. Model: The distances to the observation points are large compared to the size of the dipole, so model the field as that of a dipole moment.

Visualize:



The dipole consists of charges $\pm q$ along the y-axis. The electric field in (a) points down. The field in (b) points up.

Solve: **(a)** The dipole moment is

$$\vec{p} = (qs,\ \text{from} - \text{to} +) = (1.0\times10^{-9}\ \text{C})(0.0020\ \text{m})\hat{j} = 2.0\times10^{-12}\,\hat{j}\ \text{C m}$$

The electric field at (10 cm, 0 cm), which is at distance r = 0.10 m in the plane perpendicular to the electric dipole, is

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3} = -(9.0\times10^9\ \text{N m}^2/\text{C}^2)\frac{2.0\times10^{-12}\,\hat{j}\ \text{C m}}{(0.10\ \text{m})^3} = -18.0\hat{j}\ \text{N/C}$$

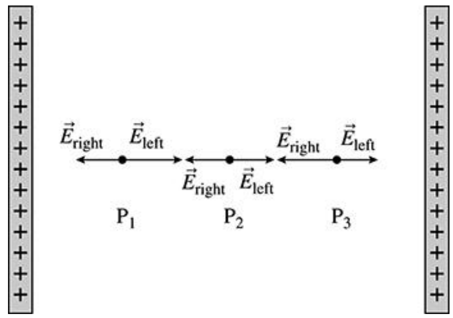
The field strength, which is all we're asked for, is 18.0 N/C.

(b) The electric field at (0 cm, 10 cm), which is at r = 0.10 m along the axis of the dipole, is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3} = (9.0\times10^9\ \text{N m}^2/\text{C}^2)\frac{2(2.0\times10^{-12}\,\hat{j}\ \text{C m})}{(0.10\ \text{m})^3} = 36\hat{j}\ \text{N/C}$$

The field strength at this point is 36 N/C.

27.9. Model: The rods are thin. Assume that the charge lies along a *line*.
Visualize:



Because both the rods are positively charged, the electric field from each rod points away from the rod. Because the electric fields from the two rods are in opposite directions at P_1, P_2, and P_3, the net field strength at each point is the difference of the field strengths from the two rods.
Solve: Example 27.3 gives the electric field strength in the plane that bisects a charged rod:

$$E_{rod} = \frac{1}{4\pi\varepsilon_0} \frac{|Q|}{r\sqrt{r^2 + (L/2)^2}}$$

The electric field from the rod on the right at a distance of 1 cm from the rod is

$$E_{right} = (9.0\times10^9 \text{ N m}^2/\text{C}^2)\frac{10\times10^{-9} \text{ C}}{(0.01 \text{ m})\sqrt{(0.01 \text{ m})^2 + (0.05 \text{ m})^2}} = 1.765\times10^5 \text{ N/C}$$

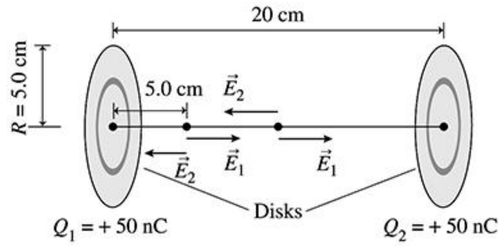
The electric field from the rod on the right at distances 2 cm and 3 cm from the rod are 0.835×10^5 N/C and 0.514×10^5 N/C. The electric fields produced by the rod on the left at the same distances are the same. Point P_1 is 1.0 cm from the rod on the left and is 3.0 cm from the rod on the right. Because the electric fields at P_1 have opposite directions, the net electric field strengths are

At 1.0 cm $\quad E = 1.765\times10^5 \text{ N/C} - 0.514\times10^5 \text{ N/C} = 1.25\times10^5 \text{ N/C}$

At 2.0 cm $\quad E = 0.835\times10^5 \text{ N/C} - 0.835\times10^5 \text{ N/C} = 0 \text{ N/C}$

At 3.0 cm $\quad E = 1.765\times10^5 \text{ N/C} - 0.514\times10^5 \text{ N/C} = 1.25\times10^5 \text{ N/C}$

27.14. Model: Model each disk as a uniformly charged disk. When the disk is positively charged, the on-axis electric field of the disk points away from the disk.
Visualize:



Solve: (a) The surface charge density on the disk is

$$\eta = \frac{Q}{A} = \frac{Q}{\pi R^2} = \frac{50\times10^{-9}\ \text{C}}{\pi(0.050\ \text{m})^2} = 6.366\times10^{-6}\ \text{C/m}^2$$

From Equation 27.23, the electric field of the left disk at $z = 0.10$ m is

$$(E_1)_z = \frac{\eta}{2\varepsilon_0}\left[1-\frac{1}{\sqrt{1+R^2/z^2}}\right] = \frac{6.366\times10^{-6}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2\text{/N m}^2)}\left[1-\frac{1}{\sqrt{1+(0.050\ \text{m}/0.10\ \text{m})^2}}\right] = 38{,}000\ \text{N/C}$$

Hence, $\vec{E}_1 = (38{,}000\ \text{N/C, right})$. Similarly, the electric field of the right disk at $z = 0.10\,\text{m}$ (to its left) is $\vec{E}_2 = (38{,}000\ \text{N/C, left})$. The net field at the midpoint between the two disks is $\vec{E} = \vec{E}_1 + \vec{E}_2 = 0\ \text{N/C}$.
(b) The electric field of the left disk at $z = 0.050$ m is

$$(E_1)_z = \frac{6.366\times10^{-6}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2\text{/N m}^2)}\left[1-\frac{1}{\sqrt{1+(0.050\ \text{m}/0.10\ \text{m})^2}}\right] = 1.05\times10^5\ \text{N/C} \Rightarrow \vec{E}_1 = (1.05\times10^5\ \text{N/C, right})$$

Similarly, the electric field of the right disk at $z = 0.15$ m (to its left) is $\vec{E}_2 = (1.85\times10^4\ \text{N/C, left})$. The net field is thus

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = (8.7\times10^4\ \text{N/C, right})$$

The field strength is 8.7×10^4 N/C.

27.19. Model: The electric field in a region of space between the plates of a parallel-plate capacitor is uniform.
Solve: The electric field inside a capacitor is $E = Q/\varepsilon_0 A$. Thus, the charge needed to produce a field of strength E is

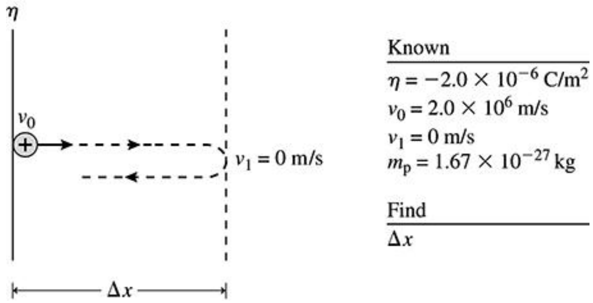
$$Q = \varepsilon_0 AE = (8.85\times10^{-12}\ \text{C}^2\text{/N m}^2)\left[\pi(0.020\ \text{m})^2\right](3.0\times10^6\ \text{N/C}) = 33.4\ \text{nC}$$

The number of electrons transferred from one plate to the other is

$$\frac{33.4\times10^{-9}\ \text{C}}{1.60\times10^{-19}\ \text{C}} = 2.1\times10^{11}$$

27.24. Model: The infinite negatively charged plane produces a uniform electric field that is directed toward the plane.

Visualize:



Solve: From the kinematic equation of motion $v_1^2 = 0 = v_0^2 + 2a\Delta x$ and $F = qE = ma$,

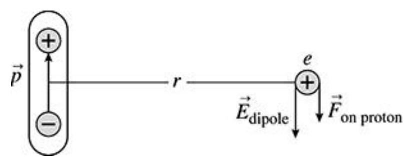
$$a = \frac{qE}{m} = \frac{-v_0^2}{2\Delta x} \Rightarrow \Delta x = \frac{-mv_0^2}{2qE}$$

Furthermore, the electric field of a plane of charge with surface charge density η is $E = \eta/2\varepsilon_0$. Thus,

$$\Delta x = \frac{-mv_0^2\varepsilon_0}{q\eta} = \frac{-\left(1.67\times10^{-27}\text{ kg}\right)\left(2.0\times10^{6}\text{ m/s}\right)^2\left(8.85\times10^{-12}\text{ C}^2/\text{N m}^2\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(-2.0\times10^{-6}\text{ C/m}^2\right)} = 0.185\text{ m}$$

27.27. Model: The size of a molecule is ≈ 0.1 nm. The proton is 2.0 nm away, so $r >> s$ and we can use Equation 27.12 for the electric field in the plane that bisects the dipole.

Visualize:



Solve: You can see from the diagram that $\vec{F}_{\text{dipole on proton}}$ is opposite to the direction of $\vec{p}$. The magnitude of the dipole field at the position of the proton is

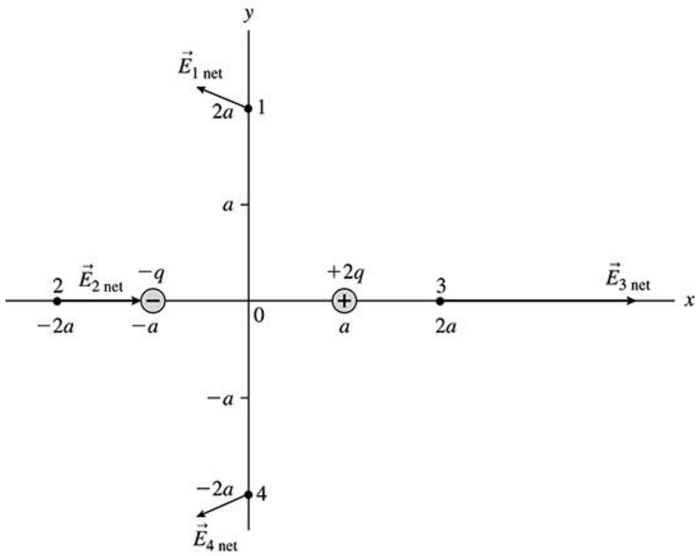
$$E_{\text{dipole}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} = (9.0\times10^9\text{ N m}^2/\text{C}^2)\frac{5.0\times10^{-30}\text{ C m}}{(2.0\times10^{-9}\text{ m})^3} = 5.624\times10^5\text{ N/C}$$

The magnitude of $\vec{F}_{\text{dipole on proton}}$ is

$$F_{\text{dipole on proton}} = eE_{\text{dipole}} = (1.60\times10^{-19}\text{ C})(5.624\times10^5\text{ N/C}) = 9.0\times10^{-13}\text{ N}$$

Including the direction, the force is $\vec{F}_{\text{dipole on proton}} = (9.0\times10^{-13}$ N, direction opposite $\vec{p}$).

27.32. Model: The electric field is that of two charges $-q$ and $+2q$ located at $x = \pm a$.
Visualize:



Solve: **(a)** At point 1, the electric field from $-q$ is

$$E_{-q} = \frac{1}{4\pi\varepsilon_0}\frac{|-q|}{(a)^2 + (2a)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{5a^2}$$

$\vec{E}_{-q}$ points toward $-q$ and makes an angle $\phi_1 = \tan^{-1}(2a/a) = 63.43°$ below the $-x$-axis, hence

$$\vec{E}_{-q} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{5a^2}\right)\left(-\cos\phi_1\hat{i} - \sin\phi_1\hat{j}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{5a^2}\right)\left(-\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{5\sqrt{5}a^2}\right)\left(-\hat{i} - 2\hat{j}\right)$$

The electric field from the $+2q$ is

$$E_{+2q} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{a^2 + (2a)^2} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{5a^2}$$

$\vec{E}_{+2q}$ points away from $+2q$ and makes an angle $\phi_1 = \tan^{-1}(2a/a) = 63.43°$ above the $-x$-axis. So,

$$\vec{E}_{+2q} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{5a^2}\right)\left(-\cos\phi_2\hat{i} + \sin\phi_2\hat{j}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{5\sqrt{5}a^2}\right)\left(-2\hat{i} + 4\hat{j}\right)$$

Adding these two vectors,

$$\vec{E}_{1\text{ net}} = \vec{E}_{-q} + \vec{E}_{+2q} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{5\sqrt{5}a^2}\right)\left(-3\hat{i} + 2\hat{j}\right)$$

At point 2, the electric field from $-q$ points toward $-q$, so

$$\vec{E}_{-q} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{a^2}\right)\hat{i}$$

The electric field from $+2q$ points away from $+2q$, so

$$\vec{E}_{+2q} = -\frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{9a^2}\right)\hat{i}$$

Adding these two vectors,

27.43. Model: The electric field is that of a line charge of length L.
Visualize: Please refer to Figure P27.43. Let the bottom end of the rod be the origin of the coordinate system. Divide the rod into many small segments of charge Δq and length $\Delta y'$. Segment i creates a small electric field at the point P that makes an angle θ with the horizontal. The field has both x and y components, but E_z = 0 N/C. The distance to segment i from point P is $\left(x^2+y'^2\right)^{1/2}$.
Solve: The electric field created by segment i at point P is

$$\vec{E}_i = \frac{\Delta q}{4\pi\varepsilon_0\left(x^2+y'^2\right)}\left(\cos\theta\hat{\imath}-\sin\theta\,\hat{\jmath}\right) = \frac{\Delta q}{4\pi\varepsilon_0\left(x^2+y'^2\right)}\left(\frac{x}{\sqrt{x^2+y'^2}}\hat{\imath}-\frac{y'}{\sqrt{x^2+y'^2}}\hat{\jmath}\right)$$

The net field is the sum of all the $\vec{E}_i$, which gives $\vec{E}=\sum_i \vec{E}_i$. Δq is not a coordinate, so before converting the sum to an integral we must relate charge Δq to length $\Delta y'$. This is done through the linear charge density $\lambda = Q/L$, from which we have the relationship

$$\Delta q = \lambda\Delta y' = \frac{Q}{L}\Delta y'$$

With this charge, the sum becomes

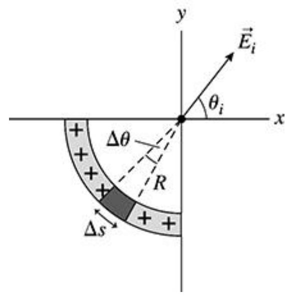
$$\vec{E} = \frac{Q/L}{4\pi\varepsilon_0}\sum_i\left[\frac{x\Delta y'}{\left(x^2+y'^2\right)^{3/2}}\hat{\imath}-\frac{y'\Delta y'}{\left(x^2+y'^2\right)^{3/2}}\hat{\jmath}\right]$$

Now we let $\Delta y' \to dy'$ and replace the sum by an integral from $y'=0$ m to $y'=L$. Thus,

$$\vec{E} = \frac{(Q/L)}{4\pi\varepsilon_0}\left(\int_0^L \frac{x\,dy'}{\left(x^2+y'^2\right)^{3/2}}\hat{\imath}-\int_0^L\frac{y'dy'}{\left(x^2+y'^2\right)^{3/2}}\hat{\jmath}\right) = \frac{(Q/L)}{4\pi\varepsilon_0}\left(x\left[\frac{y'}{x^2\sqrt{x^2+y'^2}}\right]_0^L\hat{\imath}-\left[\frac{-1}{\sqrt{x^2+y'^2}}\right]_0^L\hat{\jmath}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0}\frac{1}{x\sqrt{x^2+L^2}}\hat{\imath} - \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{Lx}\right)\left(1-\frac{x}{\sqrt{x^2+L^2}}\right)\hat{\jmath}$$

27.47. Model: Assume that the quarter-circle plastic rod is thin and that the charge lies along the quarter-circle of radius R.
Visualize:



The origin of the coordinate system is at the center of the circle. Divide the rod into many small segments of charge Δq and arc length Δs.

Solve: (a) Segment i creates a small electric field $\vec{E}_i$ at the origin with two components:

$$(E_i)_x = E_i\cos\theta_i \qquad (E_i)_y = E_i\sin\theta_i$$

Note that the angle θ_i depends on the location of the segment i. Now all segments are at distance $r_i = R$ from the origin, so

$$E_i = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{r_i^2} = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{R^2}$$

The linear charge density of the rod is $\lambda = Q/L$, where L is the rod's length (L = quarter-circumference = $\pi R/2$). This allows us to relate charge Δq to the arc length Δs through

$$\Delta q = \lambda\Delta s = \left(\frac{Q}{L}\right)\Delta s = \left(\frac{2Q}{\pi R}\right)\Delta s$$

Using $\Delta s = R\Delta\theta$, the components of the electric field at the origin are

$$(E_i)_x = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{R^2}\cos\theta_i = \frac{1}{4\pi_0}\frac{1}{R^2}\left(\frac{2Q}{\pi R}\right)R\Delta\theta\cos\theta_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{\pi R^2}\right)\Delta\theta\cos\theta_i$$

$$(E_i)_y = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q}{R^2}\sin\theta_i = \frac{1}{4\pi\varepsilon_0}\frac{1}{R^2}\left(\frac{2Q}{\pi R}\right)R\Delta\theta\sin\theta_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{\pi R^2}\right)\Delta\theta\sin\theta_i$$

(b) The x- and y-components of the electric field for the entire rod are the integrals of the expressions in part (a) from
$\theta = 0$ rad to $\theta = \pi/2$. We have

$$E_x = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{\pi R^2}\right)\int_0^{\pi/2}\cos\theta\, d\theta \qquad E_y = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{\pi R^2}\right)\int_0^{\pi/2}\sin\theta\, d\theta$$

(c) The integrals are

$$\int_0^{\pi/2}\sin\theta d\theta = [-\cos\theta]_0^{\pi/2} = -\left(\cos\frac{\pi}{2} - \cos 0\right) = +1 \qquad \int_0^{\pi/2}\cos\theta d\theta = [\sin\theta]_0^{\pi/2} = \sin\frac{\pi}{2} - \sin 0 = +1$$

The electric field is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{\pi R^2}\left(\hat{i} + \hat{j}\right)$$

27.53. Model: Assume that the electric field inside the capacitor is constant, so constant-acceleration kinematic equations apply.

Visualize: Please refer to Figure P27.53.

Solve: **(a)** The force on the electron inside the capacitor is

$$\vec{F} = m\vec{a} = q\vec{E} \Rightarrow \vec{a} = \frac{q\vec{E}}{m}$$

Because $\vec{E}$ is directed upward (from the positive plate to the negative plate) and $q = -1.60\times10^{-19}\ \text{C}$, the acceleration of the electron is downward. We can therefore write the above equation as simply $a_y = qE/m$. To determine E, we must first find a_y. From kinematics,

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2 \Rightarrow 0.040\ \text{m} = 0\ \text{m} + v_0\cos 45°(t_1 - t_0) + 0\ \text{m}$$

$$\Rightarrow (t_1 - t_0) = \frac{(0.040\ \text{m})}{(5.0\times10^6\ \text{m/s})\cos 45°} = 1.1314\times10^{-8}\ \text{s}$$

Using the kinematic equations for the motion in the y-direction,

$$v_{1y} = v_{0y} + a_y\left(\frac{t_1 - t_0}{2}\right) \Rightarrow 0\ \text{m/s} = v_0\sin 45° + \left(\frac{qE}{m}\right)\left(\frac{t_1 - t_0}{2}\right)$$

$$\Rightarrow E = -\frac{2\ \text{m}\ v_0\sin 45°}{q(t_1 - t_0)} = -\frac{2(9.1\times10^{-31}\ \text{kg})(5.0\times10^6\ \text{m/s})\sin 45°}{(-1.60\times10^{-19}\ \text{C})(1.1314\times10^{-8}\ \text{s})} = 3550\ \text{N/C} = 3.6\times10^3\ \text{N/C}$$

(b) To determine the separation between the two plates, we note that $y_0 = 0$ m and $v_{0y} = (5.0\times10^6\ \text{m/s})\sin 45°$, but at $y = y_1$, the electron's highest point, $v_{1y} = 0$ m/s. From kinematics,

$$v_{1y}^2 = v_{0y}^2 + 2a_y(y_1 - y_0) \Rightarrow 0\ \text{m}^2/\text{s}^2 = v_0^2\sin^2 45° + 2a_y(y_1 - y_0)$$

$$\Rightarrow (y_1 - y_0) = -\frac{v_0^2\sin^2 45°}{2a_y} = -\frac{v_0^2}{4a_y}$$

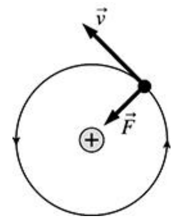
From part (a),

$$a_y = \frac{qE}{m} = \frac{(-1.60\times10^{-19}\ \text{C})(3550\ \text{N/C})}{9.1\times10^{-31}\ \text{kg}} = -6.242\times10^{14}\ \text{m/s}^2$$

$$\Rightarrow y_1 - y_0 = -\frac{(5.0\times10^6\ \text{m/s})^2}{4(-6.242\times10^{14}\ \text{m/s}^2)} = 0.010\ \text{m} = 1.0\ \text{cm}$$

This is the height of the electron's trajectory, so the minimum spacing is 1.0 cm.

27.58. Model: The electron orbiting the proton experiences a force given by Coulomb's law.
Visualize:



Solve: The force that causes the circular motion is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_\mathrm{p}|q_\mathrm{e}|}{r^2} = \frac{m_\mathrm{e}v^2}{r} = \frac{m_\mathrm{e}\left(4\pi^2r^2f^2\right)}{r}$$

where we used $v = 2\pi r/T = 2\pi rf$. The frequency is

$$f = \sqrt{\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_\mathrm{p}|q_\mathrm{e}|}{4\pi^2m_\mathrm{e}r^3}} = \sqrt{\frac{\left(9.0\times10^9\ \mathrm{N\,m^2/C^2}\right)\left(1.60\times10^{-19}\ \mathrm{C}\right)^2}{4\pi^2\left(9.11\times10^{-31}\ \mathrm{kg}\right)\left(5.3\times10^{-11}\ \mathrm{m}\right)^3}} = 6.56\times10^{15}\ \mathrm{Hz}$$

27.3. $E_3 = E_4 > E_2 > E_1$. The electric field strength is larger in the region where the field lines are closer together (E_3 and E_4) and smaller where the field lines are farther apart.

27.8. (a) $A_f = \dfrac{A_i}{3.163^2}$, so $\dfrac{N_f}{N_i} = \dfrac{(Q/A_f)}{(Q/A_i)} = \dfrac{A_i}{A_f} = 3.163^2 = 10$

(b) $F_f = eE_f = e\dfrac{N_f}{2E_0}$ and $F_i = eE_i = e\dfrac{N_i}{2E_0}$, so $\dfrac{F_f}{F_i} = \dfrac{N_f}{N_i} = 10.$